**C 40190** **(Pages : 4)** **Name..................................**

**Reg. No...............................**

## SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2023

Mathematics

MAT 6B 13(E02)—LINEAR PROGRAMMING

(2017—2018 Admissions)

Time : Three Hours Maximum Marks : 80

### Section A

*Answer* **all** *questions.*
*Each question carries* 1 *mark.*

1. What is a slack variable ?
2. What is a degenerate solution of an L.P.P. ?
3. What is the difference between a feasible solution and a basic feasible solution of an L.P.P. ?
4. Define a convex set.
5. Define a hyperplane in $\mathbb{R}^n$.
6. Write the names of any two methods to solve a transportation problem.
7. Write the following L.P.P. in standard form :

   Minimise $Z = 4x_1 - x_2 + x_3$

   subject to

   $$x_1 + x_2 - x_3 \geq 1$$
   $$4x_1 + x_2 + x_3 \leq 1$$
   $$x_1, x_2 x_3 \geq 0.$$

8. Show that $x_1 = 1, x_2 = 1, x_3 = 0$ is a feasible solution of the L.P.P. :

   Maximize $Z = 4x_1 + x_2$

   subject to $x_1 + x_2 + x_3 \leq 4$
   $$x_1 - x_2 + x_3 \geq 0$$
   $$x_1, x_2, x_3 \geq 0.$$

9. Write the necessary and sufficient condition for a basic feasible solution to a L.P.P. to be an optimum (maximum).
10. When we say that an 'Assignment problem' is unbalanced ?
11. What is degeneracy in a transportation problem ?
12. Explain why we not use 'Transportation Algorithm' to solve an 'Assignment Problem'.

(12 × 1 = 12 marks)

**Turn over**

## Section B

*Answer any* **nine** *questions.*
*Each question carries 2 marks.*

13. Write a short note on 'North-West Corner Rule'.

14. Write a short note on 'The Hungerian Method'.

15. What are 'Unbalanced Transportation Problems' ? How are they solved ?

16. Write the dual of the following L.P.P. :

$$\text{Maximise } Z = 3x_1 + 2x_2$$
$$\text{s.t. } x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 2$$
$$x_1, x_2 \ge 0.$$

17. Show that the following L.P.P. has an unbounded solution :

$$\text{Maximise } Z = 4x_1 + x_2$$
$$\text{s.t. } x_1 + x_2 \ge 1$$
$$x_1 \ge 2$$
$$x_2 \le 1$$
$$x_1, x_2 \ge 0.$$

18. Show that intersection of two convex set is convex.

19. Find a basic feasible solution of the following transportation problem by using North-West Corner Rule :

| | $D_1$ | $D_2$ | $D_3$ | |
|---|---|---|---|---|
| $O_1$ | 2 | 7 | 4 | 5 |
| $O_2$ | 3 | 3 | 1 | 8 |
| $O_3$ | 5 | 4 | 7 | 7 |
| $O_4$ | 1 | 6 | 2 | 14 |
| | 7 | 9 | 18 | |

20. Write $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ as a linear combination of $B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

21. Show that the set $A = \{(x_1, x_2, x_3) | x_1 + x_2 - x_3 = 0\}$ is a convex set.

22. Given that the vectors $V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $V_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$ and $V_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ are linearly dependent. Find the value of 'a'.

23. Show that there exists only a finite number of basic feasible solutions to a L.P.P.

24. Solve the Assignment problem :

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 1 | 2 | 3 |
| $A_2$ | 2 | 1 | 3 |
| $A_3$ | 3 | 2 | 1 |

(9 × 2 = 18 marks)

## Section C

*Answer any* **six** *questions.*
*Each question carries* 5 *marks.*

25. Solve :

$$\text{Maximise } Z = 3x_1 + 4x_2$$
$$\text{subject to } 4x_1 + 2x_2 \leq 80$$
$$2x_1 + 5x_2 \leq 180$$
$$x_1, x_2 \geq 0.$$

26. Use Simplex method to solve the following L.P.P. :

$$\text{Maximise } Z = 7x_1 + 5x_2$$
$$\text{subject to } x_1 + 2x_2 \leq 6$$
$$4x_1 + 3x_2 \leq 12$$
$$x_1, x_2 \geq 0.$$

27. Show that the following L.P.P. has no solution :

$$\text{Maximise } Z = 4x_1 + x_2$$
$$\text{subject to } x_1 + x_2 \geq 1$$
$$2x_1 - 2x_2 \leq 1$$
$$x_1 \geq 5$$
$$x_1, x_2 \geq 0.$$

28. Find an initial basic feasible solution using Vold's method to the following transportation problem :

| | I | II | III | IV | |
|---|---|---|---|---|---|
| A | 21 | 16 | 15 | 3 | 11 |
| B | 17 | 18 | 14 | 23 | 13 |
| C | 32 | 27 | 18 | 41 | 19 |
| | 6 | 10 | 12 | 15 | |

**Turn over**

29. Solve the following Assignment problem :

| | | Man A | B | C | D |
|---|---|---|---|---|---|
| | I | 12 | 30 | 21 | 15 |
| Job | II | 18 | 33 | 9 | 31 |
| | III | 44 | 25 | 24 | 21 |
| | IV | 23 | 30 | 28 | 14 |

30. Show that a hyperplane is a convex set.

31. Show that $H = \left\{x \in \mathbb{R}^n / C \times \geq z, \text{ for } C \in \mathbb{R}^n \text{ and } Z \in \mathbb{R}\right\}$ is a convex set.

32. Write the steps to solve an L.P.P. by using Simplex method.

33. Prove that dual of the dual is the primal.

(6 × 5 = 30 marks)

## Section D

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

34. Prove that a hyperplane is a closed set.

35. Solve the Assignment problem given below :

| | I | II | III | IV | V | VI |
|---|---|---|---|---|---|---|
| A | 9 | 22 | 58 | 11 | 19 | 27 |
| B | 43 | 78 | 72 | 50 | 63 | 48 |
| C | 41 | 28 | 91 | 37 | 45 | 33 |
| D | 74 | 42 | 27 | 49 | 39 | 32 |
| E | 36 | 11 | 57 | 22 | 25 | 18 |
| F | 3 | 56 | 53 | 31 | 17 | 28 |

36. Use Simplex method to solve :

Maximise $Z = 107x_1 + x_2 + 2x_3$

subject to the constraints

$$14x_1 + x_2 - x_3 + 3x_4 = 7$$

$$16x_1 + \tfrac{1}{2}x_2 - 2x_3 \leq 5$$

$$3x_1 \geq 0$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

(2 × 10 = 20 marks)