

D 103063

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Name.....

Reg. No.....

**FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2024**

Mathematics

MTS4C04—MATHEMATICS—4

(2019 Admission onwards)

Time : Two Hours

Maximum : 60 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Ceiling is 20.

1. Solve $dx + e^{3x} dy = 0$.
2. Find general solution of $x \frac{dy}{dx} - 4y = x^6 e^x$.
3. Solve the initial value problem $\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1-x^2)}$, $y(0) = 2$.
4. Determine whether the functions $f_1(x) = x$, $f_2(x) = x^2$, $f_3(x) = 4x - 3x^2$ are linearly dependent or linearly independent on the interval $(-\infty, \infty)$.
5. The function $y_1 = \ln x$ is a solution of $xy'' + y' = 0$. Find a second solution $y_2(x)$.
6. Solve the initial value problem $4y'' + 4y' + 17y = 0$, $y(0) = -1$, $y'(0) = 2$.
7. Find a homogeneous linear differential equation with constant co-efficient whose solution is $y = c_1 \cos 8x + c_2 \sin 8x$.
8. Let $f(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 1, & t \geq 1 \end{cases}$. Find $\mathcal{L}(f(t))$.

Turn over

9. Find $\mathcal{L}^{-1}\left(\frac{4s}{4s^2 + 1}\right)$.
10. Evaluate $\mathcal{L}(t \sin kt)$.
11. Show that the functions $f_1(x) = x$, $f_2(x) = \cos 2x$ are orthogonal on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
12. Show that the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - 6\frac{\partial u}{\partial y} = 0$ is parabolic.

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 30.

13. Solve $(x^2 + y^2) dx + (x^2 - xy) dy = 0$.
14. Solve the initial value problem $\frac{dy}{dx} = \cos(x + y)$, $y(0) = \frac{\pi}{4}$.
15. Solve $y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}$.
16. Solve the homogeneous boundary value problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(L) = 0$.
17. Solve $x^2 y'' - 3xy' + 3y = 2x^4 e^x$.
18. Solve $f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau) e^{t-\tau} d\tau$ for $f(t)$.
19. Expand $f(x) = x^2$, $0 < x < L$ in a cosine series.

Section C

*Answer any **one** questions.*

Each question carries 10 marks.

20. Solve the initial value problem $y'' + 4y' + 6y = 1 + e^{-t}$, $y(0) = 0$, $y'(0) = 0$ using Laplace transform.

21. Find the Fourier series of $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2, & 0 \leq x < \pi \end{cases}$ on the interval $[-\pi, \pi]$.