D 12050 **(Pages : 2)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019–2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted.

Section A (Short Answer Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Find the mean of a uniform random variable with possible values 1,2,3,4 and 5.
2. Define negative binomial distribution.
3. Obtain the m.g.f. of exponential distribution.
4. If X follow normal distribution with mean 10 and variance 9, find (i) $P(X > 13)$; (ii) $P(7 < X < 13)$.
5. Define parameter and statistic.
6. Define convergence in distribution.
7. State Central Limit theorem.
8. Define the terms (i) population ; (ii) sampling frame.
9. Define simple random sampling.
10. Find the probability that the sample mean of a random sample of size 16 taken from a normal population with mean 2 and variance 4 is less than 1.
11. If X and Y are independent standard normal random variables, identify the probability distribution of $\left[\frac{X-Y}{X+Y}\right]^2$.
12. Define t-distribution.

(8 × 3 = 24 marks)

Turn over

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. If the $(r-1)^{th}$, r^{th}, and $(r+1)^{th}$ central moments of X following binomial distribution with parameters n and p are, μ_{r-1}, μ_{r-1} and μ_r, show that $\mu_{r+1} = pq\left[nr\,\mu_{r-1} + \frac{d}{dp}\mu_r\right]$.

14. State and prove lack of memory property of exponential distribution.

15. State and prove Chebyshev's inequality.

16. Examine whether Weak Law of Large Numbers hold for the sequence of random variable $\{X_i\}$, where $P\left(X_i = \pm\sqrt{2i-1}\right) = \frac{1}{2}$.

17. State and prove Bernoulli's law of large numbers.

18. Explain cluster sampling.

19. A random sample of size 10 is taken from a normal population with mean 10 and unknown variance. If the sample variance are is 18.23, find the probabilities of the sample mean (i) less than 9 ; (ii) greater than 11.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (i) Show that odd order central moments of X following $N\left(\mu, \sigma^2\right)$ are zeroes.

    (ii) Prove that the mean deviation about the mean of X following $N\left(\mu, \sigma^2\right)$ is $\sqrt{\frac{2}{\pi}}\,\sigma$.

21. (i) Define Chi-square distribution. Obtain the m.g.f., of X following $\chi^2_{(n)}$.

    (ii) State and prove the additive property of Chi-square distribution.

(1 × 11 = 11 marks)