D 10668 (Pages : 3) Name.....

Reg. No.....

# FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

## Section A

*Answer at least* **eight** *questions.*
*Each question carries* 3 *marks.*
*All questions can be attended.*
*Overall Ceiling* 24.

1. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in [1, 2].
2. Determine fixed points of the function $g(x) = x^2 - 2$.
3. Write the equation of Lagrange's interpolating polynomial through $(x_0, y_0)$ and $(x_1, y_1)$.
4. State three point end point formula of differentiation.
5. Using Trapezoidal rule find $\int_0^2 x^2\, dx$.
6. Show that $f(t, y) = t|y|$ satisfies a Lipschitz condition on the interval D = $\{(t, y)/1 \leq t \leq 2$ and $-3 \leq y \leq 4\}$.
7. Define a convex set.
8. For all $x \geq -1$ and any positive $m$ show that $0 \leq (1 + x)^m \leq e^{mx}$.
9. When is the initial value problem $\frac{dy}{dt} = f(t, y)$, $a \leq t \leq b$, $y(a) = \alpha$ well posed.
10. What is the degree of accuracy or precision of a quadrature formula ?

**Turn over**

11. Write Newton's Forward difference formula.

12. Set up Newton-Raphson formula for computing $\sqrt{N}$.

(8 × 3 = 24 marks)

## Section B

*Answer at least* **five** *questions.*
*Each question carries* 5 *marks.*
*All questions can be attended.*
*Overall Ceiling* 25.

13. Find a root of $f(x) = x^3 - 3x - 5 = 0$ correct to 3 decimal places using Newton-Raphson method. Start with $x_0 = 3$.

14. Using Lagrange's interpolation formula find $y(10)$ if :

| | | | | | |
|---|---|---|---|---|---|
| $x$ | : | 5 | 6 | 9 | 11 |
| $y$ | : | 12 | 13 | 14 | 16 |

15. Using Newton's forward interpolation formula find the cubic polynomial for the data :

| | | | | | |
|---|---|---|---|---|---|
| $x$ | : | 0 | 1 | 2 | 3 |
| $y$ | : | 1 | 2 | 1 | 10 |

16. Approximate $\int_1^2 \frac{1}{x}\,dx$ using Simpson's $\frac{3}{8}^{\text{th}}$ rule with step value $h = 0.25$

17. Using Second derivative midpoint formula approximate $f^{11}(1.3)$ if $f(x) = 3xe^x - \cos x$ with $h = 0.1$. Given :

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | : | 1.2 | 1.29 | 1.30 | 1.31 | 1.40 |
| $y$ | : | 11.59006 | 13.78176 | 14.04276 | 14.30741 | 16.86187 |

18. Use Euler's method to find approximate solution for the initial value problem $y^1 = 1 + \frac{y}{t}$, $1 \le t \le 2, y(1) = 2$ with $h = 0.25$.

19. Use Newton's Backward difference formula to construct interpolating polynomial of degree 1 if $f(-0.75) = -.07181250$, $f(-0.5) = -.02475000$, $f(-.25) = .33493750$, $f(0) = 1.10100000$.

(5 × 5 = 25 marks)

## Section C

*Answer any* **one** *question.*
*The question carries* 11 *marks.*

20. Find by the method of Regula Falsi a root of the equation $x^3 + x^2 - 3x - 3 = 0$ lying between 1 and 2.

21. Use the Modified Euler method to approximate the solutions to the IVP $y^1 = \frac{1+t}{1+y}, 1 \le t \le 2,$ $y(1) = 2$ with $h = 0.5$.

(1 × 11 = 11 marks)